Einstein-Podolsky-Rosen entanglement for self-interacting complex scalar fields

2001 J. Phys. A: Math. Gen. 34 1853

(http://iopscience.iop.org/0305-4470/34/9/304)

J. Phys. A: Math. Gen. **34** (2001) 1853–1859 www.iop.org/Journals/ja PII: S0305-4470(01)16764-8

Einstein–Podolsky–Rosen entanglement for self-interacting complex scalar fields

Hong-Yi Fan[1,2] **and Zeng-Bing Chen**[3,4]

[1] CCAST (World Laboratory), PO Box 8730, Beijing 100080, People's Republic of China
[2] Department of Material Science and Engineering, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China[5]
[3] Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China

E-mail: zbchen@ustc.edu.cn

Received 30 August 2000

Abstract

We demonstrate the Einstein–Podolsky–Rosen (EPR) correlations in the Gaussian trial wavefunctional widely used in the Schrödinger representation of the self-interacting complex scalar fields (CSF). This demonstration is based on the explicit eigenstates $\|\xi\rangle$, which possess the precise EPR entanglement and constitute a complete and orthonormal representation, of CSF ϕ and $\phi^{\dagger}$ in the Fock space. To show the entanglement of the Gaussian trial wavefunctional, we evaluate in the $\langle\xi\|$-representation its 'entangled' Wigner functional resembling the Wigner function of the two-mode squeezed vacuum state, whose quantum entanglement is well known.

PACS numbers: 0365B, 1110

In 1935, Einstein, Podolsky and Rosen (EPR) published a famous paper [1] arguing the incompleteness of quantum mechanics. Their paper has started a lasting debate about the completeness and the meaning of local realities in quantum mechanics. The EPR paper introduced two striking aspects of quantum mechanics into physics: quantum entanglement [2] and quantum nonlocality (quantum nonseparability), though they found them to be unbelievable. The relationship between quantum entanglement and quantum nonlocality has then been a source of great theoretical interest and plays an essential role in the modern understanding of quantum phenomena. Based on Bell's work [3] and experiments [4] inspired by his work, the EPR entanglement now becomes a peculiar but basic feature of quantum mechanics and is the source of the weirdness of quantum mechanics [5]. In the burgeoning field of quantum information theory, the EPR entanglement is also of practical importance. As

[4] Author to whom correspondence should be addressed.
[5] Mailing address.

a valuable resource, the EPR entangled states can be exploited to perform classically impossible tasks, such as quantum teleportation [6, 7] and quantum computation [8].

However, as far as we know, little attention has been paid to the EPR entanglement in the context of quantum field theories. In this paper we aim to demonstrate the EPR correlations in the Gaussian trial wavefunctional [9, 10], which was widely used in the Schrödinger representation of the nonlinear model field theories, such as the $\lambda\phi^4$-model for complex scalar fields (CSF). The Schrödinger representation in quantum field theories [11, 12] is an attractive setting in the research of many structural features of various quantum field theories and the nonperturbative calculations in strong-interaction theory. It is a field-theoretical generalization of the Schrödinger equation in quantum mechanics. Thus the experience gained in quantum mechanics will help us to research and develop quantum field theories in an intuitive way. For instance, the (nonperturbative) Gaussian effective potential approach [9, 10] used in this context is analogous to the variational approach in ordinary quantum mechanics. Meanwhile the Schrödinger representation has been proved to be rather tractable in the kinematical calculations [13]. It also has various usages in many other topics, such as the topological effects [14] and confinement [15] in gauge theories, the collective phenomena [16, 17] and quantization of (1 + 1)-dimensional gravity [18].

The Lagrangian density for the CSF with self-interaction is

$$\mathcal{L}(x) = -\partial_\mu \phi^\dagger \partial_\mu \phi - m^2 \phi^\dagger \phi - U(\phi^\dagger \phi). \tag{1}$$

The Hamiltonian operator

$$H = \int \mathrm{d}x^3 \, [\Pi^\dagger \Pi + (\nabla \phi^\dagger) \cdot (\nabla \phi) + m^2 \phi^\dagger \phi + U(\phi^\dagger \phi)]. \tag{2}$$

In quantum theory of the CSF, ϕ and $\phi^\dagger$ are two independent fields. Henceforth we work in the Schrödinger picture, for example, $\phi(x) \equiv \phi(\boldsymbol{x}, t = 0)$. The following commutation relations are nonvanishing:

$$\begin{aligned} &[\phi(x), \Pi(x')] = \mathrm{i}\delta(\boldsymbol{x} - \boldsymbol{x}') \\ &[\phi^\dagger(x), \Pi^\dagger(x')] = \mathrm{i}\delta(\boldsymbol{x} - \boldsymbol{x}') \end{aligned} \tag{3}$$

where the conjugate fields $\Pi(x) = \partial_t \phi^\dagger(x)$ and $\Pi^\dagger(x) = \partial_t \phi(x)$. Choosing $U(\phi^\dagger\phi) = \frac{1}{6}\lambda_0(\phi^\dagger\phi)^2$, one obtains the $\lambda\phi^4$-model for the CSF. Note that this model is important in the Goldstone–Higgs mechanism [19, 20], a cornerstone of the standard model.

Obviously, in the Schrödinger representation for the quantized $\phi^\dagger$ and ϕ, one needs a *complete* basis $\|\xi\rangle$ in which the field operators $\phi^\dagger(x)$ and $\phi(x)$ are diagonal; the state vectors $\|\Psi(t)\rangle$ of the CSF in the Schrödinger representation read $\langle\xi\|\Psi(t)\rangle \equiv \Psi[\xi, t]$. As comparison, recall that in ordinary quantum mechanics, the coordinate operator Q is diagonal in the coordinate basis $|q\rangle$ and the wavefunctions $\psi(q, t)$ are the inner products between the state vectors $|\psi(t)\rangle$ and $|q\rangle$. Apparently, the $\langle\xi\|$-representation is a necessary kinematical framework for the Schrödinger representation of the CSF. The explicit construction of the $\langle\xi\|$-representation is lacking from the literature, at least to our knowledge.

Very recently, the explicit common eigenstates $\|\xi\rangle$ of $\phi(x)$ and $\phi^\dagger(x)$ were constructed in the Fock space [21]. Here we briefly review the construction of $\|\xi\rangle$. The CSF can be canonically quantized and divided into a positive frequency part and a negative frequency part as [22]

$$\phi(\boldsymbol{x}, t) = \phi_+ + \phi_- \qquad \phi^\dagger(\boldsymbol{x}, t) = \phi_+^\dagger + \phi_-^\dagger \tag{4}$$

where

$$\begin{aligned} \phi_+ &= \sum_{\boldsymbol{p}} a_{\boldsymbol{p}} f_{\boldsymbol{p}}(\boldsymbol{x}, t) = (\phi_-^\dagger)^\dagger \\ \phi_- &= \sum_{\boldsymbol{p}} b_{\boldsymbol{p}}^\dagger f_{\boldsymbol{p}}^*(\boldsymbol{x}, t) = (\phi_+^\dagger)^\dagger \end{aligned} \tag{5}$$

with $[a_{\boldsymbol{p}}, a^{\dagger}_{\boldsymbol{p}'}] = [b_{\boldsymbol{p}}, b^{\dagger}_{\boldsymbol{p}'}] = \delta_{\boldsymbol{p}\boldsymbol{p}'}$, $f_{\boldsymbol{p}}(\boldsymbol{x}, t) = (2V\omega_{\boldsymbol{p}})^{-1/2}\mathrm{e}^{\mathrm{i}(\boldsymbol{p}\cdot\boldsymbol{x}-\omega_{\boldsymbol{p}}t)}$, $\omega_{\boldsymbol{p}} = \sqrt{m^2+\boldsymbol{p}^2}$ and V being the normalization volume. For the self-interacting fields m is an arbitrary real mass parameter [22], and one may choose m to be the physical mass m_{phys}. Since $\phi(x)$ and $\phi^{\dagger}(x)$ are commutative and independent of each other, one can construct their common eigenstates, which in the Fock space are given by

$$\|\xi\rangle = \left(\det\frac{G}{2}\right)^{-\frac{1}{2}} \exp\left\{\iint \mathrm{d}^3x\,\mathrm{d}^3x'\,G^{-1}(\boldsymbol{x}-\boldsymbol{x}')[-\xi^*(\boldsymbol{x})\xi(\boldsymbol{x}') - 2\phi^{\dagger}_{-}(x)\phi_{-}(x') + 2\xi(\boldsymbol{x})\phi^{\dagger}_{-}(x') + 2\xi^*(\boldsymbol{x})\phi_{-}(x')]\right\}\|00\rangle. \tag{6}$$

Here the vacuum state of the CSF is annihilated by both $a_{\boldsymbol{p}}$ and $b_{-\boldsymbol{p}}$:

$$a_{\boldsymbol{p}}|00\rangle_{\boldsymbol{p}} = b_{-\boldsymbol{p}}|00\rangle_{\boldsymbol{p}} = 0 \qquad \|00\rangle = \prod_{\boldsymbol{p}}|00\rangle_{\boldsymbol{p}} \tag{7}$$

and

$$G^{-1}(\boldsymbol{x}-\boldsymbol{x}') = \sum_{\boldsymbol{p}}\frac{\omega_{\boldsymbol{p}}}{V}\mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{x}')} \tag{8}$$

is the inverse of $G(\boldsymbol{x}-\boldsymbol{x}') = \sum_{\boldsymbol{p}}\frac{\mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{x}')}}{\omega_{\boldsymbol{p}}V}$ due to the following fact:

$$\int \mathrm{d}^3y\,G(\boldsymbol{x}-\boldsymbol{y})G^{-1}(\boldsymbol{y}-\boldsymbol{x}') = \delta(\boldsymbol{x}-\boldsymbol{x}'). \tag{9}$$

It can be proved that [21]

$$\phi(x)\|\xi\rangle = \xi(\boldsymbol{x})\|\xi\rangle \qquad \phi^{\dagger}(x)\|\xi\rangle = \xi^*(\boldsymbol{x})\|\xi\rangle. \tag{10}$$

Therefore $\|\xi\rangle$ are indeed the common eigenstates of ϕ and $\phi^{\dagger}$. Physically, the field 'configurations' $\xi(\boldsymbol{x})$ ($\xi^*(\boldsymbol{x})$), being the eigenvalues of $\phi(x)$ ($\phi^{\dagger}(x)$), should be smooth and complex functions of the space coordinates $\boldsymbol{x}$. Moreover, $\|\xi\rangle$ satisfy the completeness and orthonormality relations:

$$\int\left[\frac{\mathrm{d}^2\xi}{\pi}\right]\|\xi\rangle\langle\xi\| = 1 \tag{11}$$

$$\langle\xi' \,\|\, \xi\rangle = [\pi]\delta^{(2)}[\xi'-\xi]. \tag{12}$$

Here the integral in equation (11) is of course the functional one [12], and $\delta^{(2)}[\xi = \xi_1 + \mathrm{i}\xi_2] = \delta[\xi_1]\delta[\xi_2]$ is the functional δ-function of complex arguments ξ. Equations (11) and (12) imply that $\|\xi\rangle$ form a complete and orthonormal representation, the $\langle\xi\|$-representation. The common eigenvectors $\|\eta\rangle$ of $\Pi(x)$ and $\Pi^{\dagger}(x)$ can be similarly obtained. The inner products between $\|\xi\rangle$ and $\|\eta\rangle$ are

$$\langle\xi\|\eta\rangle = \mathrm{e}^{\mathrm{i}\int\mathrm{d}^3x\,[\xi^*(\boldsymbol{x})\eta^*(\boldsymbol{x})+\xi(\boldsymbol{x})\eta(\boldsymbol{x})]} = (\langle\eta\|\xi\rangle)^*. \tag{13}$$

This is consistent with the fact that in the $\langle\xi\|$-representation,

$$\Pi(x) = -\mathrm{i}\frac{\delta}{\delta\xi(\boldsymbol{x})} \qquad \Pi^{\dagger}(x) = -\mathrm{i}\frac{\delta}{\delta\xi^*(\boldsymbol{x})} \tag{14}$$

and

$$\begin{aligned}\langle\xi\|\Pi(x)\|\eta\rangle &= -\mathrm{i}\frac{\delta}{\delta\xi(\boldsymbol{x})}\langle\xi\|\eta\rangle = \eta(\boldsymbol{x})\langle\xi\|\eta\rangle \\ \langle\xi\|\Pi^{\dagger}(x)\|\eta\rangle &= -\mathrm{i}\frac{\delta}{\delta\xi^*(\boldsymbol{x})}\langle\xi\|\eta\rangle = \eta^*(\boldsymbol{x})\langle\xi\|\eta\rangle.\end{aligned} \tag{15}$$

Remarkably, $\|\xi\rangle$ possess the precise EPR entanglement. In EPR's original argument, they introduced the EPR states, the common eigenstates $|\eta\rangle$ of the relative position $Q_1 - Q_2$ and the total momentum $P_1 + P_2$ of two particles [1]. Their explicit forms in two-mode Fock space were given only recently [23]. Another set of the EPR states, the common eigenstates $|\xi\rangle$ of compatible operators $(Q_1 + Q_2, P_1 - P_2)$, can also be explicitly constructed in two-mode Fock space [23]:

$$|\xi\rangle = \mathrm{e}^{-\frac{1}{2}|\xi|^2 + \xi a_1^\dagger + \xi^* a_2^\dagger - a_1^\dagger a_2^\dagger}|00\rangle \tag{16}$$

which describe a perfectly correlated two-particle system. Here $[a_j, a_k^\dagger] = \delta_{jk}$, $a_j + a_j^\dagger = \sqrt{2}Q_j$, $a_j - a_j^\dagger = \mathrm{i}\sqrt{2}P_j$ $(j = 1, 2)$ and $|00\rangle$ is the two-mode vacuum state. In fact, $\|\xi\rangle$ are the field-theoretical generalization of the EPR entangled states $|\xi\rangle$. Using the Fourier transformation $\xi(\boldsymbol{x}) = \frac{1}{\sqrt{V}}\sum_{\boldsymbol{p}} \xi_{\boldsymbol{p}} \mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{x}}$, $\|\xi\rangle$ can be rewritten in the momentum space as

$$\|\xi\rangle = \prod_{\boldsymbol{p}} \sqrt{2\omega_{\boldsymbol{p}}} \exp\left(-\omega_{\boldsymbol{p}}|\xi_{\boldsymbol{p}}|^2 + \sqrt{2\omega_{\boldsymbol{p}}}\xi_{\boldsymbol{p}} a_{\boldsymbol{p}}^\dagger + \sqrt{2\omega_{\boldsymbol{p}}}\xi_{\boldsymbol{p}}^* b_{-\boldsymbol{p}}{}^\dagger - a_{\boldsymbol{p}}^\dagger b_{-\boldsymbol{p}}^\dagger\right)\|00\rangle. \tag{17}$$

The simultaneous appearance of both $a_{\boldsymbol{p}}^\dagger$ and $b_{-\boldsymbol{p}}^\dagger$ originates from the conservation of momentum.

It is obvious that the $\boldsymbol{p}$-mode component of $\|\xi\rangle$ in equation (17) is equivalent to the EPR entangled states $|\xi\rangle$ defined in equation (16) and exhibits the entanglement between the positively and negatively charged quanta of the CSF. Therefore the newly constructed eigenstates $\|\xi\rangle$ are the field-theoretical generalization of $|\xi\rangle$. In this sense one can call $\|\xi\rangle$ the entangled eigenstates of the CSF. It is worthwhile to point out that the entangled eigenstates in equation (6) or (17) are compatible with the superselection rule [24]. The $\|\xi\rangle$ states are obtained by both $a_{\boldsymbol{p}}^\dagger$ and $b_{-\boldsymbol{p}}^\dagger$ acting on the vacuum state (see equation (17)). The two operators ($a_{\boldsymbol{p}}^\dagger$ and $b_{-\boldsymbol{p}}^\dagger$) create the positively and negatively charged quanta simultaneously from the vacuum, thus ensuring the conservation law of charge.

The $\langle\xi\|$-representation has an immediate application to the Schrödinger representation in the nonlinear model for the CSF under study. Now the Schrödinger representation is realized in a Hilbert space, a linear space of wavefunctionals $\Psi[\xi, t] = \langle\xi\|\Psi(t)\rangle$, where the argument runs over all smooth field configurations $\xi(\boldsymbol{x})$ and the *time-independent* field operators $\phi(\boldsymbol{x})$ and $\phi^\dagger(\boldsymbol{x})$ are diagonal:

$$\begin{aligned} \langle\xi\|\phi(x)\|\Psi(t)\rangle &\equiv \phi(x)\Psi[\xi, t] = \xi(\boldsymbol{x})\Psi[\xi, t] \\ \langle\xi\|\phi^\dagger(x)\|\Psi(t)\rangle &\equiv \phi^\dagger(x)\Psi[\xi, t] = \xi^*(\boldsymbol{x})\Psi[\xi, t] \end{aligned} \tag{18}$$

as is evident from equation (10). The corresponding conjugate field operators become

$$\begin{aligned} \Pi(\boldsymbol{x})\Psi[\xi, t] &= -\mathrm{i}\frac{\delta}{\delta\xi(\boldsymbol{x})}\Psi[\xi, t] \\ \Pi^\dagger(\boldsymbol{x})\Psi[\xi, t] &= -\mathrm{i}\frac{\delta}{\delta\xi^*(\boldsymbol{x})}\Psi[\xi, t] \end{aligned} \tag{19}$$

as can be seen from equation (14). In this way, the commutation relations (3) are fulfilled, and the Hamiltonian (2) can then be written in the Schrödinger representation as

$$H = \int \mathrm{d}^3x \left[-\frac{\delta^2}{\delta\xi^*\delta\xi} + |\boldsymbol{\nabla}\xi|^2 + m^2|\xi|^2 + U(|\xi|^2)\right]. \tag{20}$$

The Schrödinger equation turns out to be

$$\mathrm{i}\frac{\partial}{\partial t}\Psi[\xi, t] = H\Psi[\xi, t] \tag{21}$$

which takes the form of a functional partial differential equation, similar to the conventional Schrödinger equation in quantum mechanics.

Since $\|\xi\rangle$ are precisely the field EPR entangled states, the $\langle\xi\|$-representation is useful in revealing the EPR entanglement for a *physical* field state (note that $\|\xi\rangle$ are normalized to a δ-functional and thus unphysical). To show this, we make the following ansatz for the field state (the variational vacuum state):

$$\begin{aligned}\Psi[\xi, \varpi, F] &= N_F \exp\left\{-\tfrac{1}{2}\iint \mathrm{d}^3x\, \mathrm{d}^3y\, [\xi^*(\boldsymbol{x}) - \varpi^*(\boldsymbol{x})]F(\boldsymbol{x}-\boldsymbol{y})[\xi(\boldsymbol{y}) - \varpi(\boldsymbol{y})]\right\}\\ &= \langle\xi\|\Psi[\varpi, F]\rangle \end{aligned} \tag{22}$$

where $N_F = \sqrt{\det F}$ is a normalization constant and the real function $F(\boldsymbol{x}-\boldsymbol{y}) = F(\boldsymbol{y}-\boldsymbol{x})$. The Gaussian trial wavefunctional (22) was widely used in the Gaussian effective potential approach [9, 10], specifically to the $\lambda\phi^4$-model [10]. Using the completeness relation (11) of $\|\xi\rangle$, we obtain

$$\|\Psi[\varpi, F]\rangle = \int \left[\frac{\mathrm{d}^2\xi}{\pi}\right] \Psi[\xi, \varpi, F]\|\xi\rangle. \tag{23}$$

In [25], the authors introduced the 'entangled Wigner operator' for two-mode correlated systems. Here we present its generalization to the CSF in the $\langle\xi\|$-representation. Such generalization is necessary in revealing the EPR correlations of field states for the CSF. Let A stand for an operator which is also a function or functional of the field variables. Then the Weyl correspondence rule [26] is

$$A = \int [\mathrm{d}^2\gamma\, \mathrm{d}^2\lambda]\Delta(\gamma, \lambda)\mathcal{A}(\gamma, \lambda) \tag{24}$$

where the entangled Wigner operator $\Delta(\gamma, \lambda)$ in the $\langle\xi\|$-representation is

$$\Delta(\gamma, \lambda) \equiv [4]\int \left[\frac{\mathrm{d}^2\xi}{\pi^3}\right] \| - \xi + \gamma\rangle\langle\xi + \gamma\|\mathrm{e}^{-2\mathrm{i}\int \mathrm{d}^3x\, [\xi^*(\boldsymbol{x})\lambda^*(\boldsymbol{x}) + \xi(\boldsymbol{x})\lambda(\boldsymbol{x})]}. \tag{25}$$

Therefore, equation (24) maps, via $\Delta(\gamma, \lambda)$, a classical quantity $\mathcal{A}$ into a quantum mechanical operator A.

To show the entanglement of the Gaussian trial wavefunctional (22), we evaluate in the $\langle\xi\|$-representation its 'entangled Wigner functional' from equations (12) and (25):

$$\begin{aligned}W_{\Psi[\varpi,F]} &= \langle\Psi[\varpi, F]\|\Delta(\gamma, \lambda)\|\Psi[\varpi, F]\rangle\\ &= [4]\int \left[\frac{\mathrm{d}^2\xi}{\pi^3}\right] \Psi^*[\gamma - \xi, \varpi, F]\Psi[\gamma + \xi, \varpi, F]\mathrm{e}^{-2\mathrm{i}\int \mathrm{d}^3x\, [\xi^*(\boldsymbol{x})\lambda^*(\boldsymbol{x}) + \xi(\boldsymbol{x})\lambda(\boldsymbol{x})]}. \end{aligned} \tag{26}$$

Substituting equation (22) into (26) yields

$$\begin{aligned}W_{\Psi[\varpi,F]}(\gamma, \lambda) = \left[\frac{4}{\pi^2}\right]\exp\bigg\{ -\iint \mathrm{d}^3x\, \mathrm{d}^3y\, &(4\lambda^*(\boldsymbol{x})F^{-1}(\boldsymbol{x}-\boldsymbol{y})\lambda(\boldsymbol{y})\\ &+[\gamma^*(\boldsymbol{x}) - \varpi^*(\boldsymbol{x})]F(\boldsymbol{x}-\boldsymbol{y})[\gamma(\boldsymbol{y}) - \varpi(\boldsymbol{y})])\bigg\} \end{aligned} \tag{27}$$

where $F^{-1}(\boldsymbol{x}-\boldsymbol{y})$ is the inverse of $F(\boldsymbol{x}-\boldsymbol{y})$:

$$\int \mathrm{d}^3y\, F(\boldsymbol{x}-\boldsymbol{y})F^{-1}(\boldsymbol{y}-\boldsymbol{x}') = \delta(\boldsymbol{x}-\boldsymbol{x}'). \tag{28}$$

Making use of the Fourier transformations of $\lambda(x)$, $\varpi(x)$, $\gamma(x)$ and

$$F(x-y)=\frac{1}{\sqrt{V}}\sum_p F_p e^{ip\cdot(x-y)}$$
$$F^{-1}(x-y)=\frac{1}{\sqrt{V}}\sum_p F_p^{-1} e^{ip\cdot(x-y)} \tag{29}$$

with $F_p^{-1}=1/F_p$ due to equation (28), the Wigner functional $W_{\Psi[\varpi,F]}(\gamma,\lambda)$ in the momentum space reads

$$W_{\Psi[\varpi,F]}(\gamma,\lambda)=\prod_p W_{\Psi[\varpi_p,F_p]}(\gamma_p,\lambda_p)$$
$$W_{\Psi[\varpi_p,F_p]}(\gamma_p,\lambda_p)\equiv\frac{4}{\pi^2}e^{-4|\lambda_p|^2/F_p-F_p|\gamma_p-\varpi_p|^2}. \tag{30}$$

For the purpose of illustration, one can take $\varpi_p=0$. Then setting $F_p/2=e^{-\mu_p}$, $\gamma_p=(\alpha_p-\beta_p^*)/\sqrt{2}$ and $\lambda_p=(\alpha_p+\beta_p^*)/\sqrt{2}$, $W_{\Psi[\varpi_p,F_p]}(\gamma_p,\lambda_p)$ becomes

$$W_{\Psi[\varpi_p,F_p]}(\gamma_p,\lambda_p)=\frac{4}{\pi^2}\exp[-2(|\alpha_p|^2+|\beta_p|^2)\cosh\mu_p+2(\alpha_p\beta_p+\alpha_p^*\beta_p^*)\sinh\mu_p] \tag{31}$$

which resembles the Wigner function of the two-mode squeezed vacuum state [27, 28].

It was demonstrated recently that the two-mode squeezed vacuum state exhibits striking quantum nonlocality [28]. This important state can be generated by the nondegenerate parametric amplifier and used to realize the EPR paradox in the EPR original sense [27]. The squeezed-state entanglement is also essential to the quantum teleportation of continuous variables [7]. Therefore, one may expect that the field state $\|\Psi[\varpi,F]\rangle$ also possesses EPR entanglement and quantum nonlocality. The present result also indicates that $\|\Psi[\varpi,F]\rangle$ is a squeezed state of the self-interacting CSF. The 'degree' of EPR entanglement and quantum nonlocality of the field state depends on the value of $F(x-y)$, which is determined by minimizing $\langle\Psi[\varpi,F]\|H\|\Psi[\varpi,F]\rangle$ [10]; in other words, it depends on the specific model (e.g. the $\lambda\phi^4$-model) one chooses.

To summarize, we have shown the EPR entanglement involved in the Gaussian trial wavefunctional of the nonlinear model for the CSF with self-interaction. In this paper, the $\langle\xi\|$-representation is a useful tool in uncovering the EPR entanglement for the specific field-theoretical model. So far, the EPR-type experiments mainly rely on the two-particle entanglement [4]. The EPR entanglement in the field-theoretical case, as we have demonstrated, is of conceptual importance. This intriguing phenomenon might imply that the results of measurements on the CSF at two different space points are correlated, even though the two points are *spacelike* separated. Interestingly, note also that such quantum nonlocality associated with the EPR correlations in the variational vacuum state arises here from a *local* quantum field theory. Does this imply that the vacuum 'breaks' the locality of quantum field theory in this case, just as it breaks symmetry in the Goldstone–Higgs mechanism? If not, how do we reconcile the quantum nonlocality stated above with the locality underlying quantum field theory? It seems that the EPR correlations and quantum nonlocality are more subtle in quantum field theory than in quantum mechanics and thus deserve further study.

Acknowledgments

This paper was supported by the President's Foundation of the Science Academy of China and the National Natural Science Foundation of China.

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[2] Schrödinger E 1935 *Proc. Camb. Phil. Soc.* **31** 555
Schrödinger E 1935 *Naturwissenschaften* **23** 807
[3] Bell J S 1964 *Physics (Long Island)* **1** 195
Bell J S 1987 *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press)
[4] Clauser J F and Shimony A 1978 *Rep. Prog. Phys.* **41** 1881
Aspect A, Dalibard J and Roger G 1982 *Phys. Rev. Lett.* **49** 1804
Ou Z Y and Mandel L 1988 *Phys. Rev. Lett.* **61** 50
Kwiat P G, Mattle K, Weinfurter H and Zeilinger A 1995 *Phys. Rev. Lett.* **75** 4337
[5] Knight P 1998 *Nature* **395** 12
[6] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
Bouwmeester D, Pan J-W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575
[7] Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869
Furusawa A, Sørensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
Milburn G J and Braunstein S L 1999 *Phys. Rev.* A **60** 937
[8] Steane A 1998 *Rep. Prog. Phys.* **61** 117
[9] Stevenson P M 1984 *Phys. Rev.* D **30** 1712
Ingermanson R 1986 *Nucl. Phys.* B **266** 620
Stevenson P M and Tarrach R 1986 *Phys. Lett.* B **176** 436
[10] Ni G J, Lou S Y and Chen S Q 1988 *Int. J. Mod. Phys.* A **3** 1735
Ni G J, Lou S Y and Chen S Q 1988 *Phys. Lett.* B **200** 161
Lou S Y and Ni G J 1989 *Phys. Rev.* D **40** 3040
[11] Symanzik K 1981 *Nucl. Phys.* B **190** [FS3] 1
Lüscher M 1985 *Nucl. Phys.* B **254** 52
[12] Hatfield B 1992 *Quantum Field Theory of Point Particles and Strings* (Redwood City, CA: Addison-Wesley)
[13] Floreanini R and Jackiw R 1986 *Phys. Lett.* B **175** 428
Floreanini R and Vinet L 1987 *Phys. Rev.* D **36** 1731
Floreanini R, Hill C T and Jackiw R 1987 *Ann. Phys., NY* **175** 345
[14] Treiman S, Jackiw R, Zumino B and Witten E 1985 *Current Algebra and Anomalies* (Singapore: World Scientific)
[15] Feynman R 1981 *Nucl. Phys.* B **188** 479
[16] Jackiw R 1977 *Rev. Mod. Phys.* **49** 681
[17] Rajaraman R 1982 *Solitons and Instantons* (Amsterdam: North-Holland)
[18] Benedict E, Jackiw R and Lee H-J 1996 *Phys. Rev.* D **54** 6213
[19] Goldstone J 1961 *Nuovo Cimento* **19** 154
[20] Higgs P W 1964 *Phys. Rev. Lett.* **13** 508
Higgs P W 1966 *Phys. Rev.* **145** 1156
[21] Fan H Y and Chen Z B 2000 *J. Phys. A: Math. Gen.* **33** 2145
[22] Lee T D 1981 *Particle Physics and Introduction to Field Theory* (Chur: Harwood)
[23] Fan H Y and Klauder J R 1994 *Phys. Rev.* A **49** 704
[24] Wick G C, Wightman A S and Wigner E P 1952 *Phys. Rev.* **88** 101
[25] Fan H Y and Fan Y 1998 *Mod. Phys. Lett.* A **13** 433
[26] Weyl H 1927 *Z. Phys.* **46** 1
[27] Ou Z Y, Pereira S F, Kimble H J and Peng K C 1992 *Phys. Rev. Lett.* **68** 3663
Ou Z Y, Pereira S F and Kimble H J 1992 *Appl. Phys.* B **55** 265
[28] Banaszek K and Wódkiewicz K 1998 *Phys. Rev.* A **58** 4345
Banaszek K and Wódkiewicz K 1999 *Phys. Rev. Lett.* **82** 2009